

EXERCISE III

Examples of Divisors on Algebraic Curves

1. Prove that two (different) lines X_F and X_G in $\mathbb{C}P^2$ intersect in precisely one point by computing the degree of the intersection divisor $\text{div}(G)$ on X_F .
2. Let X be the smooth projective plane curve defined by $y^2z - x^3 - xz^2$. Compute the intersection divisors $\text{div}(x)$, $\text{div}(y)$ and $\text{div}(z)$ on X .
3. Let X be the smooth projective plane conic defined by $xy = z^2$. For a fixed point $p \in X$, find all homogeneous polynomials $G(x, y, z)$ such that the associated intersection divisor is $\text{div}(G) = 2 \cdot p$.
4. Compute degree and genus of the “twisted cubic curve”, which is the smooth local intersection curve $X \subset \mathbb{C}P^3$ defined by

$$xw = yz \quad , \quad xz = y^2 \quad \text{and} \quad yw = z^2.$$

Hint: compute the intersection divisor $\text{div}(x)$ on X .

5. Let Z be a hyperelliptic curve with covering map $\zeta : Z \rightarrow \mathbb{C}_\infty$. Compute the branch divisor $B_\zeta \in \text{Div}(\mathbb{C}_\infty)$ and the ramification divisor $R_\zeta \in \text{Div}(Z)$ and show that $\zeta^*B_\zeta = 2R_\zeta$.
6. Let Z be the hyperelliptic curve defined by $y^2 = x^5 - x$. Compute the principal divisors $\text{div}(x)$ and $\text{div}(y)$.